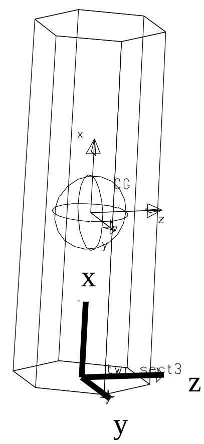
# Appendix E
# Derivation of Tapered Part Inertias

For the purposes of ADAMS/WT, we assume that a tapered PART has running (i.e. per unit length) mass and mass moments of inertia that vary linearly along the x-axis of the reference coordinate system. Note that this is *not* equivalent to linearly varying cross-section properties. An example tower section is shown here.




In addition, we assume that the offsets of the local section centroids are small compared to the PART dimensions, which is equivalent to saying that the part is beam-like. Under these assumptions we can write:

$$m(x) = m_0 + \frac{m_L - m_0}{L} x$$

$$I(x) = I_0 + \frac{I_L - I_0}{L} x, \text{ for } \mathrm{I_X, I_Y, I_Z}$$

We can easily find the total part mass by analytically integrating the running mass along the length of the PART.

$$m_{total} = \int_0^L m(x)dx = \frac{L}{2}(m_o + m_L)$$

We find the x-wise location of the center-of-gravity by balancing the first mass moments.

$$x_{CG} = \frac{L(2m_L + m_0)}{3(m_L + m_0)}$$

Taking advantage of the small offset assumption, we can also define the y and z locations of the PART's CG based on the values as the two ends of the PART.

$$y_{CG} = y_{CG_0} + \frac{(y_{CG\,L} - y_{CG_0})}{L}(x_{CG} - x_0)$$
$$z_{CG} = z_{CG_0} + \frac{(z_{CG_L} - z_{CG_0})}{L}(x_{CG} - x_0)$$

The true relation for computing any of the second mass moments should be:

$$I = \frac{I_L + I_0}{2} L + \int_0^L r^2 dm$$

We can again take advantage of the small offset assumption to get the following approximately equivalent relations for these mass moments:

$$I_X = \frac{I_{X_0} + I_{X_L}}{2} L + \int_0^L m(x)(z_{CG}(x) - z_{CG})^2 dx$$
$$I_Y = \frac{I_{Y_0} + I_{Y_L}}{2} L + \int_0^L m(x)(x - x_{CG})^2 dx$$
$$I_Z = \frac{I_{Z_0} + I_{Z_L}}{2} L + \int_0^L m(x)(x - x_{CG})^2 dx$$

$I_X$ can fortunately be analytically simplified using the assumed linear displacement of the CG along the element's length.

$$\int_0^L m(x)(z_{CG}(x) - z_{CG})^2 dx = \left(\frac{z_{CG_L} - z_{CG_0}}{L}\right)^2 \int_0^L m(x)(x - x_{CG})^2 dx$$

By substituting the first expression for *m(x)* in this integral, it can also be completed analytically to get

$$\int_0^L m(x)(x - x_{CG})^2 dx = \frac{{x_{CG}}^2}{2}(m_L + m_0)L - \frac{x_{CG}}{3}(m_L + 2m_0)L^2 + \frac{(3m_L + m_0)}{12}L^3$$

At this point we have all the necessary expressions to code the mass properties of this linearly tapered PART from the end point data, which is what is done in the ADAMS/WT macros. Again, it is important to note that the assumption of linearly-varying inertial properties does *not* correspond to a beam with uniform density and linearly-tapering cross-section (which would give quadratically varying mass, etc.) It is also not consistent with the assumption of linearly varying stiffness properties that was used in the previous discussion of tapered beam FIELD elements.

Because typical rotor blades are not of uniform density and because their radial variations are often nonlinear, this approach was developed to automatically generate blade PARTs for a *better* approximation for dynamic response than would result from using constant inertial properties along the element length. When used in conjunction with the tapered beam element, a fairly good dynamic representation of a typical rotor blade or tower is produced (usually at least as good as the input data).

If the user does need a more detailed inertial model of a blade or tower, he could generate an accurate 3-D solids CAD representation, manually divide it into pieces corresponding to ADAMS PARTs, and then use the CAD code to produce more correct inertial properties. Of course, a more complete FEA model of the structure can similarly be used to generate more precise running section properties for the FIELD stiffness matrices.